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Estimation of Structural Response Using Remote Sensor Locations

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A method is presented for estimating the response of a structure during its operation at discrete locations that are inaccessible for measurement using sensors. The prediction is based on measuring response at other locations on the structure and transforming it into the response at the desired locations using a transformation matrix. The transformation is computed using the system Markov parameters determined from a vibration test in which the response is measured both at the locations that will possess sensors during structure operation and at the desired locations that will not possess sensors. Two different approaches are considered. The first requires as many sensors as there are modes responding in the data. The second approach, a generalization of the first, only requires as many sensors as the number of input locations. The transformation matrix is shown to consist of a set of inverse system Markov parameters, which can be related to a corresponding observer formulation. A numerical example is considered using the controls-structures interaction evolutionary model testbed at NASA Langley Research Center. Acceleration response with 10% rms noise at six sensor locations is used to predict the response at four force input locations. The proposed method is not computationally intensive and, combined with the fact that the process is causal, may allow real-time applications.

Introduction

ACCURATE knowledge of response produced at discrete locations within a flexible structure during its operation is vital to many engineering applications such as control, structural monitoring, damage detection, parameter identification, and loads analysis. In the case of structural monitoring and damage detection, response is required at identified critical areas within the structure. In many instances, sensors, such as accelerometers, can be easily placed at the locations of interest. However, situations can exist in which the desired locations are not accessible for measurement during the structure's operation. This may be the case when locations of interest are at interfaces between substructures in a larger structural system.

Inverse filtering of signals[1−3] represents an instance in which collocation of inputs and outputs is desirable. A specific application of current interest is the determination of discrete input forces based on measured structural response.[4,5] In this situation, the input forces cannot be measured directly; therefore, the input locations are usually not accessible for response measurement either. Noncollocation of inputs and outputs produces a nonminimum phase system, which by definition possesses unstable zeros. Inversion of such a system produces a filter, which has unstable poles. The corresponding filter output becomes unbounded.

A method is presented by which the response at discrete inaccessible locations can be predicted using the response measured elsewhere on the structure. A transformation matrix is produced based on the unit pulse response obtained from a vibration test of the structure or substructure prior to it being placed in operation. Response measured during structural operation can then be transformed into the corresponding response at the desired locations. The transformation matrix is shown to consist of a set of inverse system Markov parameters, which can be related to a corresponding observer formulation and the Kalman filter. Note that the proposed procedure can be performed entirely using only data obtained from a straightforward vibration test.

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Description of Structural System

The structural system to be analyzed has a discrete time representation, which is governed by the difference equation

$$\boldsymbol{x}(k+1) = A\boldsymbol{x}(k) + B\boldsymbol{u}(k) \tag{1}$$

in which $\boldsymbol{x}$ represents an n-dimensional state vector, A is the $n \times n$ system matrix, B is the $n \times n_a$ input influence matrix, and $\boldsymbol{u}$ is the n_a-dimensional force vector. Index k indicates the appropriate time step. The corresponding system output equation is given by

$$\boldsymbol{y}_s(k) = C_s\boldsymbol{x}(k) + D_s\boldsymbol{u}(k) \tag{2}$$

where $\boldsymbol{y}_s$ is an n_s-dimensional sensor output vector, C_s is an $n_s \times n$ output influence matrix, and D_s is an $n_s \times n_a$ direct feedthrough matrix.

It is assumed throughout this analysis that accelerometers are used as sensors such that the direct feedthrough matrix is nonzero; however, other types of sensors can be used by appropriately modifying the derivations.[6] In the case of a modal representation of the structure given by

$$\ddot{\boldsymbol{q}} + 2\varsigma\omega\dot{\boldsymbol{q}} + \omega^2\boldsymbol{q} = \chi_a^T u \tag{3}$$

the matrices A and B will be time discretized versions of

$$A_c = \begin{bmatrix} 0 & I \\ -\omega^2 & -2\varsigma\omega \end{bmatrix} \qquad B_c = \begin{bmatrix} 0 \\ \chi_a^T \end{bmatrix} \tag{4}$$

whereas matrices C_s and D_s are of the form

$$C_s = \begin{bmatrix} -\chi_s\omega^2 & -2\chi_s\varsigma\omega \end{bmatrix} \qquad D_s = \chi_s\chi_a^T \tag{5}$$

in which ω is a diagonal matrix of modal frequencies, ς is a diagonal matrix of modal damping coefficients, and χ_s and χ_a are the mode shapes partitioned to the sensor and input locations, respectively.

For zero initial conditions, Eqs. (1) and (2) can be combined to produce the output at any time step in the form

$$\boldsymbol{y}_s(k) = \sum_{i=0}^{k} H_{si}\boldsymbol{u}(k-i) \tag{6}$$

Equation (6) represents a moving average model of the discrete system where the $n_s \times n_a$ weighting matrices H_{si}, called Markov parameters, are given by

$$H_{s0} = D_s \qquad H_{si} = C_s A^{i-1} B \qquad i = 1, 2, 3, \ldots \tag{7}$$

The Markov parameters represent the response of the discrete system to unit force pulses at the input locations and, thus, contain the dynamic properties of the structure. They can be obtained by experimentally measuring the output of the system due to a known input and computing the corresponding frequency response functions. The Markov parameters are then derived by computing the inverse discrete Fourier transform.

Prediction of Response Using as many Sensors as Modes

Assume that $\boldsymbol{y}$ represents the acceleration response due to input forces $\boldsymbol{u}$ at both the sensor and desired response locations and that it can be written in the form

$$\boldsymbol{y} = \chi \ddot{\boldsymbol{q}} \tag{8}$$

in which χ is the matrix of mode shapes that respond in the output and $\ddot{\boldsymbol{q}}$ is the vector of corresponding modal accelerations. Equation (8) can be partitioned into sensor and desired locations to produce the two relations

$$\boldsymbol{y}_s = \chi_s \ddot{\boldsymbol{q}} \tag{9}$$

$$\boldsymbol{y}_d = \chi_d \ddot{\boldsymbol{q}} \tag{10}$$

in which subscripts s and d refer to sensor and desired location partitions, respectively. Equation (9) can be solved for the modal acceleration using a least-squares approach:

$$\ddot{\boldsymbol{q}} = \left[\chi_s^T \chi_s\right]^{-1} \chi_s^T \boldsymbol{y}_s \tag{11}$$

Substitution into Eq. (10) produces

$$\hat{\boldsymbol{y}}_d = \chi_d \left[\chi_s^T \chi_s\right]^{-1} \chi_s^T \boldsymbol{y}_s = P \boldsymbol{y}_s \tag{12}$$

where $\hat{\boldsymbol{y}}_d$ represents the estimate of the acceleration response at the desired locations. Using the transformation matrix P, the desired location response can be estimated from the acceleration measured at other locations on the structure. Note that for this approach to be valid, the number of sensors n_s must be greater than or equal to the number of modes n_m that are excited by the inputs, and the modal partitions χ_s must be linearly independent. The author has developed a method, called Effective Independence, which can be used to optimally place sensors to satisfy these conditions.[7] It is proposed that mode shapes derived from a pretest finite element model can be used to optimally place sensors on the structure.

To compute the transformation matrix P using Eq. (12), the modal coefficients at both the desired and sensor locations must be known. Test-based modal coefficients can be computed from the same test data that must be obtained to compute the Markov parameters. Several methods have been developed for this purpose.[8] Note that the response of the structure must be measured during the test at both the desired response locations and the locations that will possess a sensor when the structure is in operation.

An alternative approach can be taken for computing the transformation P directly from the test data without estimating the mode shapes. Assuming that there are n_t points in the test data, there will in general be n_t Markov parameters H_i, which can also be partitioned according to sensor and desired locations as

$$H_i = \begin{bmatrix} H_{si} \\ H_{di} \end{bmatrix} \qquad i = 0, n_t - 1 \tag{13}$$

From previous discussions, it is known that the response at any location can be computed by convolving the corresponding forward system Markov parameters with the input forces. Therefore,

$$\boldsymbol{y}_s(k) = \sum_{i=0}^{k} H_{si} \boldsymbol{u}(k-i) \tag{14}$$

$$\boldsymbol{y}_d(k) = \sum_{i=0}^{k} H_{di} \boldsymbol{u}(k-i) \tag{15}$$

Equation (12) then implies

$$H_{di} = P H_{si} \qquad i = 0, n_t - 1 \tag{16}$$

However, due to the Cayley–Hamilton theorem,[9] there are at most $2n_m + 1$ independent Markov parameters. Therefore, only $N_C \geq 2n_m + 1$ equations from Eq. (16) must be considered. Assembling the sensor and desired location Markov parameters into the matrices

$$\begin{aligned} H_s &= \left[H_{s0} \quad H_{s1} \quad \cdots \quad H_{sN_C-1}\right] \\ H_d &= \left[H_{d0} \quad H_{d1} \quad \cdots \quad H_{dN_C-1}\right] \end{aligned} \tag{17}$$

and using Eq. (16) results in

$$P H_s = H_d \tag{18}$$

An estimate of P can be computed as

$$\hat{P} = H_d H_s^{\dagger} \tag{19}$$

in which $H_s^{\dagger}$ is the Moore–Penrose pseudoinverse of H_s. Note that, in general, H_s will not be full row rank. The pseudoinverse of H_s can be accurately computed, even in the presence of sensor noise, by employing the singular value decomposition.[10] The accuracy of the estimate for P can be checked using Eq. (18). Using $\hat{P}$, any response measured on the substructure at the sensor locations during vehicle operation can be used to estimate the corresponding response at the desired locations.

For the solution $\hat{P}$ in Eq. (18) to exist, block row matrix H_d must lie in the row space of H_s. This is guaranteed by the previously discussed conditions in which it is assumed that $n_s \geq n_m$ and χ_s is full column rank.

Prediction of Response Using Fewer Sensors than Modes

In some cases, there may be many modes responding in the sensor output, making the proposed method of predicting the response at the input locations cumbersome due to the large number of required sensors. This section focuses on modifying the previous method such that fewer than n_m sensors are required. It is now assumed that $n_s \leq n_m$. The idea is to still use an equation analogous to Eq. (18), such as

$$P \tilde{H}_s = H_d \tag{20}$$

but to increase the row rank of $\tilde{H}_s$ such that H_d will always lie in its row space.

The proper expression for $\tilde{H}_s$ can be found by expanding Eq. (14) for the kth and past $N_R - 1$ time steps and arranging in matrix form as

$$\begin{bmatrix} H_{s0} & H_{s1} & \cdots & \cdots & H_{sk} \\ 0 & H_{s0} & \cdots & \cdots & H_{sk-1} \\ 0 & 0 & \vdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & H_{s0} & \cdots & H_{sk-N_R+1} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}(k) \\ \boldsymbol{u}(k-1) \\ \vdots \\ \vdots \\ \boldsymbol{u}(0) \end{bmatrix} = \begin{bmatrix} \boldsymbol{y}_s(k) \\ \boldsymbol{y}_s(k-1) \\ \vdots \\ \vdots \\ \boldsymbol{y}_s(k-N_R+1) \end{bmatrix} \tag{21}$$

whereas for the kth time step, Eq. (15) is given by

$$[H_{d0} \quad H_{d1} \quad \cdots \quad H_{dk}] \begin{bmatrix} \boldsymbol{u}(k) \\ \boldsymbol{u}(k-1) \\ \vdots \\ \vdots \\ \boldsymbol{u}(0) \end{bmatrix} = \boldsymbol{y}_d(k) \tag{22}$$

Premultiplying Eq. (21) by the transformation P and utilizing Eqs. (20–22) produces

$$P\tilde{H}_s\begin{bmatrix}\boldsymbol{u}(k)\\ \boldsymbol{u}(k-1)\\ \vdots\\ \vdots\\ \boldsymbol{u}(0)\end{bmatrix} = P\begin{bmatrix}\boldsymbol{y}_s(k)\\ \boldsymbol{y}_s(k-1)\\ \vdots\\ \vdots\\ \boldsymbol{y}_s(k-N_R+1)\end{bmatrix}$$

$$= H_d\begin{bmatrix}\boldsymbol{u}(k)\\ \boldsymbol{u}(k-1)\\ \vdots\\ \vdots\\ \boldsymbol{u}(0)\end{bmatrix} = \boldsymbol{y}_d(k) \tag{23}$$

If transformation P can be found, the response at the desired locations at time k, $\boldsymbol{y}_d(k)$, can be predicted using the current sensor response $\boldsymbol{y}_s(k)$ and the response from the past $N_R - 1$ time steps.

The corresponding matrix equation that must be solved for P is given by

$$P\begin{bmatrix}H_{s0} & H_{s1} & \cdots & \cdots & H_{sN_C-1}\\ 0 & H_{s0} & \cdots & \cdots & H_{sN_C-2}\\ 0 & 0 & \vdots & \cdots & \vdots\\ \vdots & \vdots & \vdots & \cdots & \vdots\\ 0 & \cdots & H_{s0} & \cdots & H_{sN_C-N_R}\end{bmatrix}$$

$$= \begin{bmatrix}H_{d0} & H_{d1} & \cdots & H_{dN_C-1}\end{bmatrix} \tag{24}$$

As discussed earlier, solution P exists if H_d lies in the row space of $\tilde{H}_s$. Note that the order of the transformation P is given by N_R. To maximize the efficiency of the desired response prediction algorithm, N_R should be picked as small as possible. Because of its form, the maximum possible rank of $\tilde{H}_s$ is given by $N_R \cdot n_s$. It can be shown that to identify the underlying system, N_R must be selected such that $N_R \cdot n_s \geq 2n_{oc}$, where n_{oc} is the number of observable and controllable modes.[11] In practice, N_R is selected several times larger than the corresponding value. Note that sensors must be placed such that all modes observable from the desired response locations are also observable from the sensor locations.

The conditions under which $\tilde{H}_s$ attains its maximal rank can be examined by writing $\tilde{H}_s$ in the form

$$\tilde{H}_s = [H_{sR} \quad H_{sE}] \tag{25}$$

in which

$$H_{sR} = \begin{bmatrix}H_{s0} & H_{s1} & \cdots & \cdots & H_{sN_R-1}\\ 0 & H_{s0} & \cdots & \cdots & H_{sN_R-2}\\ 0 & 0 & \ddots & \cdots & \vdots\\ \vdots & \vdots & \vdots & H_{s0} & H_{s1}\\ 0 & \cdots & \cdots & 0 & H_{s0}\end{bmatrix}$$

$$H_{sE} = \begin{bmatrix}H_{sN_R} & H_{sN_R+1} & \cdots & H_{sN_C-1}\\ H_{sN_R-1} & H_{sN_R} & \cdots & H_{sN_C-2}\\ \vdots & \vdots & \cdots & \vdots\\ H_{s1} & H_{s2} & \cdots & H_{sN_C-N_R}\end{bmatrix} \tag{26}$$

Because of its structure, H_{sR} will be full column rank if and only if H_{s0} is full column rank. Therefore, to maximize the rank of $\tilde{H}_s$, sensor locations should be selected such that the system is observable and H_{s0} is full column rank. This implies that $n_s \geq n_a$, instead of the previous requirement $n_s \geq n_m$. Matrix H_{sE} can be recognized as the block Hankel matrix $H(0)$ used in the Eigensystem Realization Algorithm[8] for system identification. It can be shown that it has at most $2n_{oc}$ independent columns when N_C is selected such that $N_C - N_R \geq 2n_{oc}$ (Ref. 11). Because accelerometers are being used as sensors, any rigid body modes are not observable. If there are more sensors than inputs and if N_R is fixed, $\tilde{H}_s$ achieves maximal rank if the number of block columns is selected such that $N_C \geq N_R + 2n_{oc}$. In practice, N_C is selected much larger than this value to ensure an overdetermined system of equations.

Assuming that conditions are met such that a solution for P can be found, given n_t sensor location response data points, desired location response for the corresponding n_t time steps can be computed using

$$P\begin{bmatrix}\boldsymbol{y}_{s0} & \boldsymbol{y}_{s1} & \cdots & \cdots & \boldsymbol{y}_{sn_t-1}\\ 0 & \boldsymbol{y}_{s0} & \cdots & \cdots & \boldsymbol{y}_{sn_t-2}\\ 0 & 0 & \vdots & \cdots & \vdots\\ \vdots & \vdots & \vdots & \cdots & \vdots\\ 0 & \cdots & \boldsymbol{y}_{s0} & \cdots & \boldsymbol{y}_{sn_t-N_R}\end{bmatrix}$$

$$= \begin{bmatrix}\boldsymbol{y}_{d0} & \boldsymbol{y}_{d1} & \cdots & \boldsymbol{y}_{dn_t-1}\end{bmatrix} \tag{27}$$

Note that the desired location response at time zero depends only on the measured sensor response at zero, whereas the desired response at time $k = 1$ depends on the measured response at both $k = 1$ and $k = 0$. This pattern continues until, finally, at time $k = N_R - 1$ and beyond, the desired location response is computed using the current and past $N_R - 1$ measured sensor responses. Note that no future responses are required so that the process remains causal. It is also important to realize that there is no restriction relative to either N_C or N_R on the number of data points n_t that can be predicted for the desired location response. Whereas this procedure assumes that the system initial conditions are zero, it can also be accurately applied to systems with nonzero initial conditions to predict response for time steps $k \geq N_R$.

Relation to Inverse System Observer Markov Parameters

Equation (27) represents a moving average model of a remote sensing system with input $\boldsymbol{y}_s$ and output $\boldsymbol{y}_d$, which can be written alternatively in the form

$$\begin{aligned}\boldsymbol{y}_d(k) &= \sum_{i=0}^{k} P_i\boldsymbol{y}_s(k-i) \qquad k \leq N_R - 1\\ \boldsymbol{y}_d(k) &= \sum_{i=0}^{N_R-1} P_i\boldsymbol{y}_s(k-i) \qquad k > N_R - 1\end{aligned} \tag{28}$$

where P_i are a set of $n_d \times n_s$ Markov parameters contained in transformation matrix P identified in the preceding section. A state-space representation of this remote sensing system can be derived by solving Eq. (2) for $\boldsymbol{u}$:

$$\boldsymbol{u}(k) = -D_s^+C_s\boldsymbol{x}(k) + D_s^+\boldsymbol{y}_s(k) \tag{29}$$

and substituting into Eq. (1) and an output equation analogous to Eq. (2), but for the response at the desired locations. This produces the system

$$\boldsymbol{x}(k+1) = \hat{A}\boldsymbol{x}(k) + \hat{B}\boldsymbol{y}_s(k) \tag{30}$$

$$\boldsymbol{y}_d(k) = \hat{C}\boldsymbol{x}(k) + \hat{D}\boldsymbol{y}_s(k) \tag{31}$$

in which

$$\hat{A} = A - BD_s^+C_s \qquad \hat{B} = BD_s^+ \tag{32}$$

$$\hat{C} = C_d - D_dD_s^+C_s \qquad \hat{D} = D_dD_s^+ \tag{33}$$

and D_s^+ is the generalized inverse given by

$$D_s^+ = \left(D_s^TD_s\right)^{-1}D_s^T \tag{34}$$

Equations (30) and (31) have the desired input and output, but Eq. (30) is an inverse state equation with respect to Eq. (1) (Ref. 6), meaning that the output and input have been switched. As already discussed, the system represented by Eqs. (1) and (2) will be nonminimum phase, meaning that it possesses unstable zeros. The inverse state matrix $\hat{A}$ will then have unstable eigenvalues, meaning that Eqs. (30) and (31) cannot be used to estimate the response at the desired locations based on the measured response.

This problem can be addressed by artificially introducing damping to make the system as stable as desired using an observer formulation.[12] The artificial damping is introduced by adding and then subtracting the term $G\boldsymbol{y}_d$ in Eq. (30) where G is an arbitrary gain matrix, which can be specified to provide as much damping as desired. The resulting discrete time system equation can then be cast in the form

$$\boldsymbol{x}(k+1) = \bar{A}\boldsymbol{x}(k) + \bar{B}v(k) \tag{35}$$

in which

$$\bar{A} = \hat{A} + G\hat{C} \qquad \bar{B} = [\hat{B} + G\hat{D} \quad -G] \qquad v(k) = \begin{bmatrix} \boldsymbol{y}_s(k) \\ \boldsymbol{y}_d(k) \end{bmatrix} \tag{36}$$

Combining Eq. (35) with output Eq. (31) produces the following input–output relation for the observer-based system:

$$\begin{aligned} \boldsymbol{y}_d(k) &= \sum_{i=0}^{k} \bar{H}_i \boldsymbol{v}(k-i) \qquad & k \le p \\ \boldsymbol{y}_d(k) &= \sum_{i=0}^{p} \bar{H}_i \boldsymbol{v}(k-i) \qquad & k > p \end{aligned} \tag{37}$$

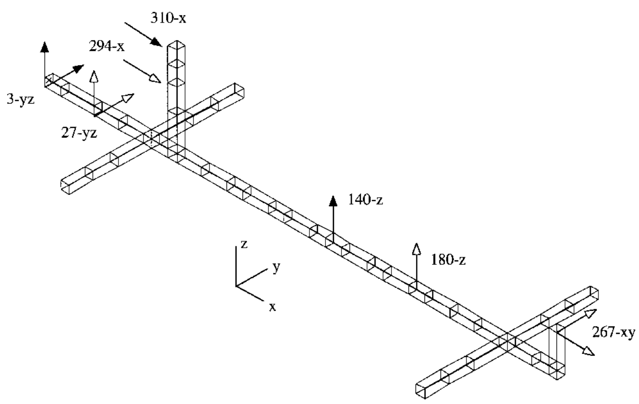

Fig. 1 CEM finite element model; solid arrows denote inputs, and open arrows denote sensors.

where it has been assumed that there are only $p+1$ observer Markov parameters given by

$$\bar{H}_0 = [\hat{D} \quad 0] \qquad \bar{H}_i = \hat{C}\bar{A}^{i-1}\bar{B} \qquad i = 1, p$$

In general, the system matrices and the gain matrix appearing in Eqs. (35) and (36) are not available for the computation of the observer Markov parameters. Rather than specifying the gain matrix G to provide the appropriate damping and decay of the observer Markov parameters, an alternate approach can be used to compute the observer Markov parameters directly from the test data such that the observer-based system becomes deadbeat after p time steps. This implies that the observer Markov parameters are zero after that time. Further details can be found in Refs. 11 and 12.

Remember that the output of the original discrete inverse system and the observer-based system are identical. The output relationship for the observer-based system given by Eq. (37) can be expanded and applied to the test-based sensor and desired location Markov parameters from Eq. (13), producing the matrix equation

$$\bar{H}V = H_d \tag{38}$$

in which

$$\bar{H} = [\hat{D} \quad \bar{H}_1 \quad \cdots \quad \bar{H}_p] \tag{39}$$

$$V = \begin{bmatrix} H_{s0} & H_{s1} & \cdots & \cdots & \cdots & H_{sN_C-1} \\ 0 & H_{s0} & H_{s1} & \cdots & \cdots & H_{sN_C-2} \\ 0 & H_{d0} & H_{d1} & \cdots & \cdots & H_{dN_C-2} \\ 0 & 0 & H_{s0} & \cdots & \cdots & \vdots \\ 0 & 0 & H_{d0} & \cdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \cdots & \cdots & \vdots \\ 0 & \cdots & \cdots & H_{s0} & \cdots & H_{sN_C-p-1} \\ 0 & \cdots & \cdots & H_{d0} & \cdots & H_{dN_C-p-1} \end{bmatrix} \tag{40}$$

A set of $p+1$ observer Markov parameters can then be estimated by solving Eq. (38) using the Moore–Penrose pseudoinverse of V.

Comparing Eqs. (24) and (40), if N_R, N_C, and sensor locations are selected according to the preceding section, and if $p = N_R - 1$, then

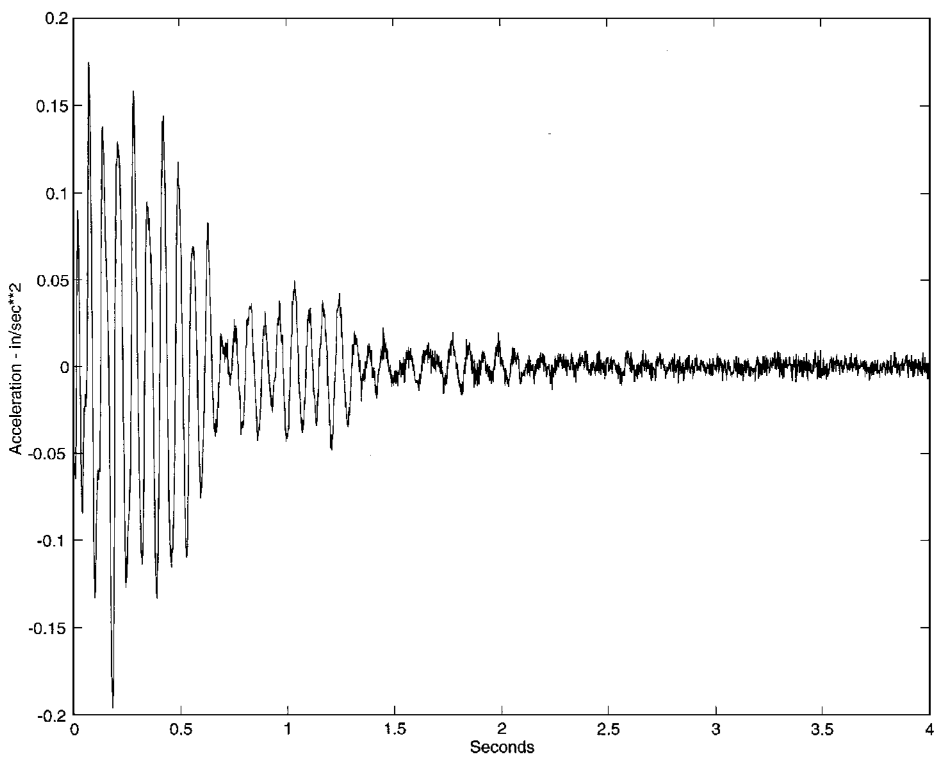

Fig. 2 Unit pulse response at 294-x due to input at 3-y.

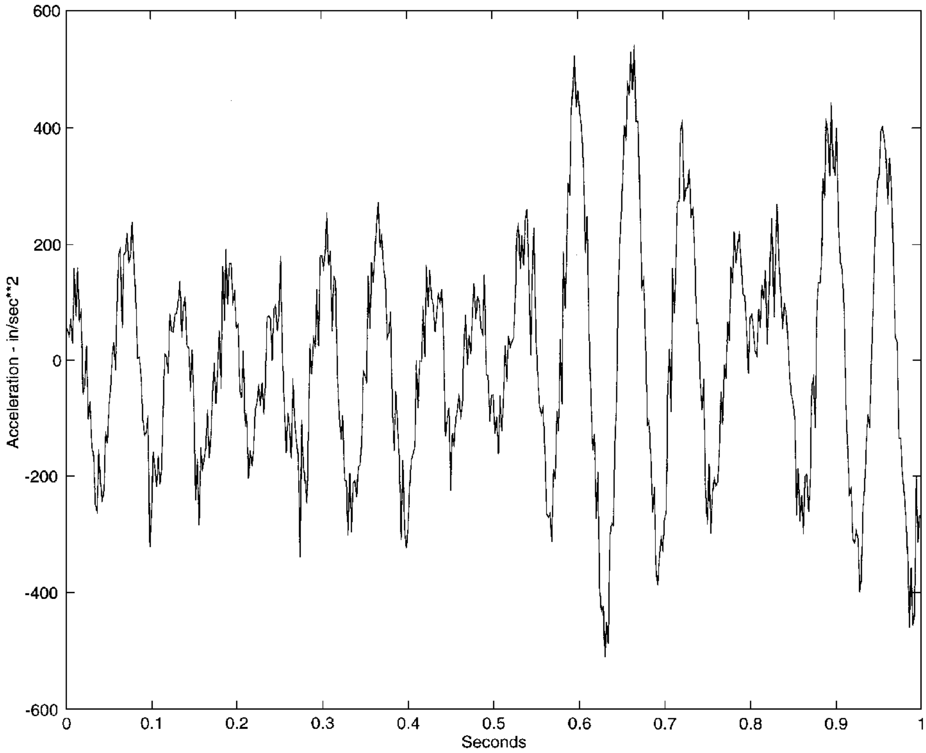

Fig. 3 Noisy response at sensor location 27-y.

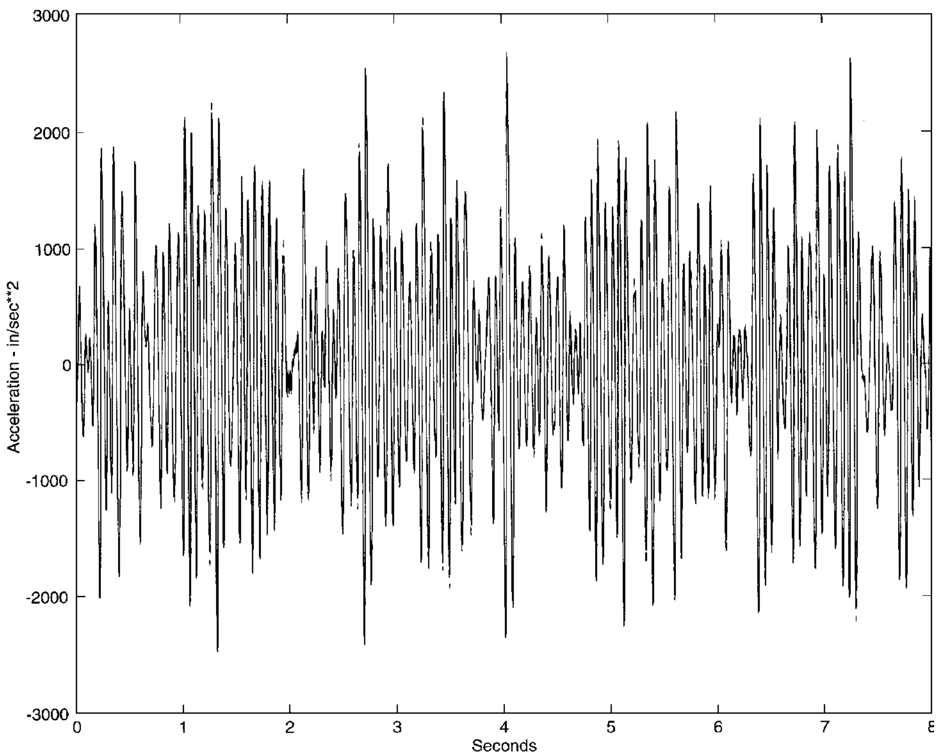

Fig. 4 Response at input location 310-x: – – –, exact and ——, predicted.

matrices V and $\tilde{H}_s$ will have the same row space. A comparison of Eqs. (20) and (38) then indicates that the estimated Markov parameters for the observer-based system and the remote sensing system discussed earlier produce equivalent input–output systems, which are deadbeat after p time steps in the absence of noise. The advantage of the proposed remote sensing formulation is that only the measured sensor response is required to estimate the structural response at the desired locations, whereas in the observer formulation, both the measured sensor response and the past estimated desired responses are required. In addition, the remote sensing formulation will, in general, result in a smaller data matrix that must be inverted in the computation.

Juang et al.[12] have shown that using the presented deterministic deadbeat observer equations on noisy data results in a Kalman filter in the limit as the amount of data used in its formulation tends to infinity. In practice, when a finite amount of data is used, the computed filter satisfies an optimality condition indicating that it is the best filter obtainable for the finite amount of data.

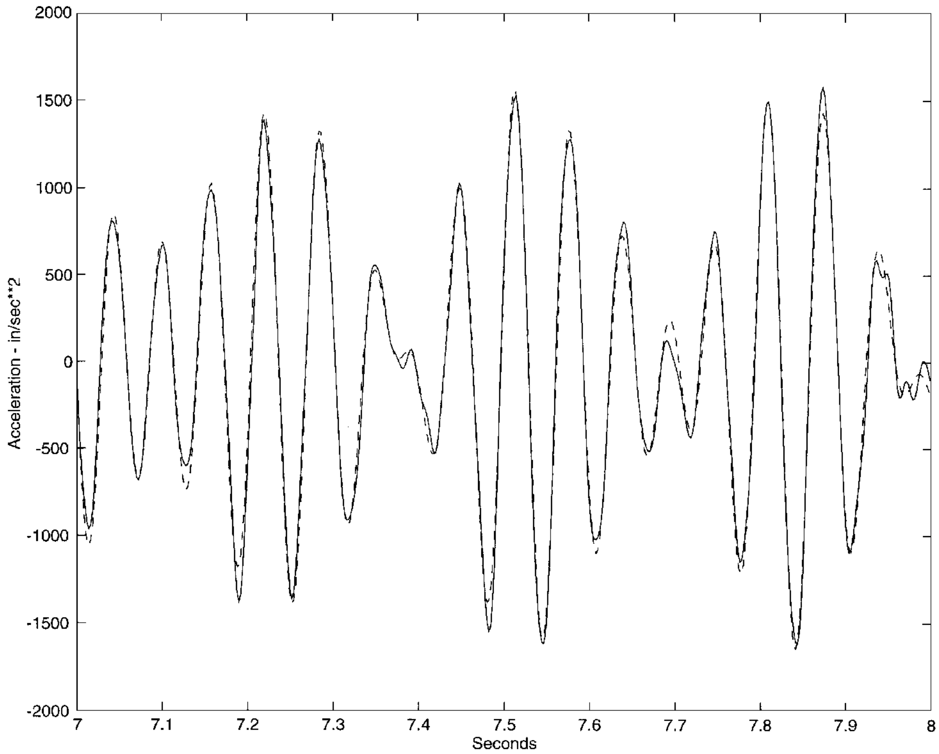

Fig. 5 Response at input location 3-y: – – –, exact and ——, predicted.

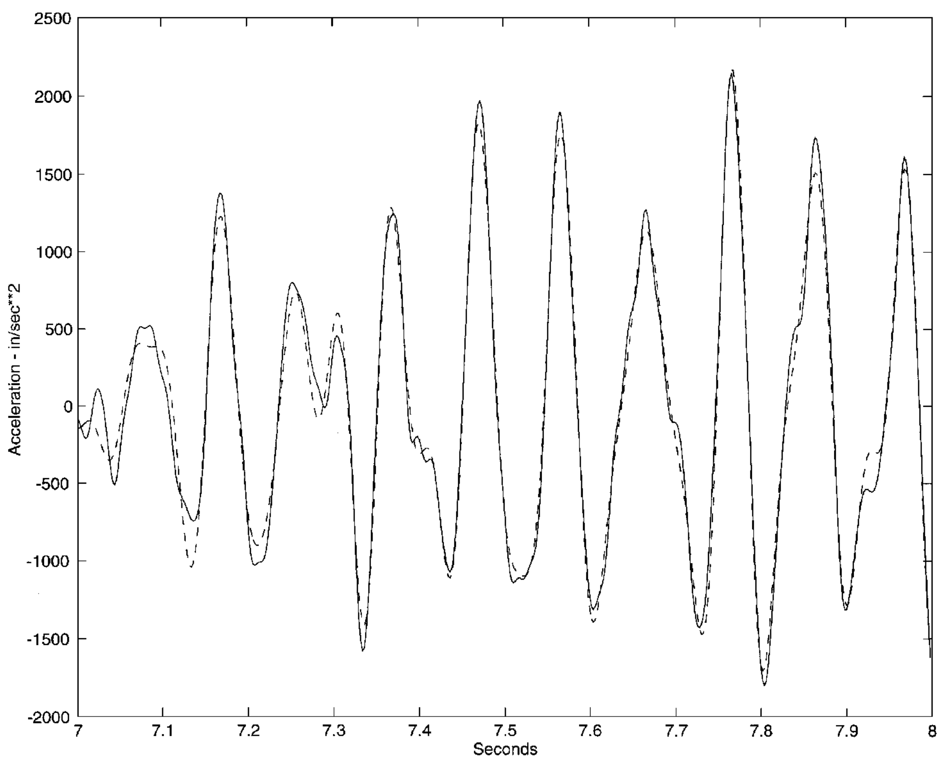

Fig. 6 Response at input location 3-z: – – –, exact and ——, predicted.

Numerical Example

This section considers the application of the proposed response estimation algorithm to the controls-structures interaction evolutionary model (CEM) testbed at NASA Langley Research Center (LaRC) Space Structures Research Laboratory. The corresponding finite element model (FEM), with much of the detail removed for clarity, is shown in Fig. 1. It contains 482 grid points with 2892 degrees of freedom. The structure is supported by two cable systems attached to the forward and aft suspension trusses. The model was reduced to a modal representation containing 27 modes in a frequency range of 0.0–35.0 Hz. Four locations on the structure were selected for inputs and six were selected for placement of accelerometers, as illustrated in Fig. 1. The objective of this example was to excite the structure, measure the response at the sensor locations, and then predict the corresponding response at the input locations. Therefore, in this case, the desired locations and the input locations are identical. However, the sensors and inputs are not collocated. This system is not minimum phase and the corresponding inverse system, Eqs. (30) and (31), is unstable.

A modal test was numerically simulated for the CEM. The 27-mode model was excited at the four input locations using random forces, and the corresponding acceleration response was measured

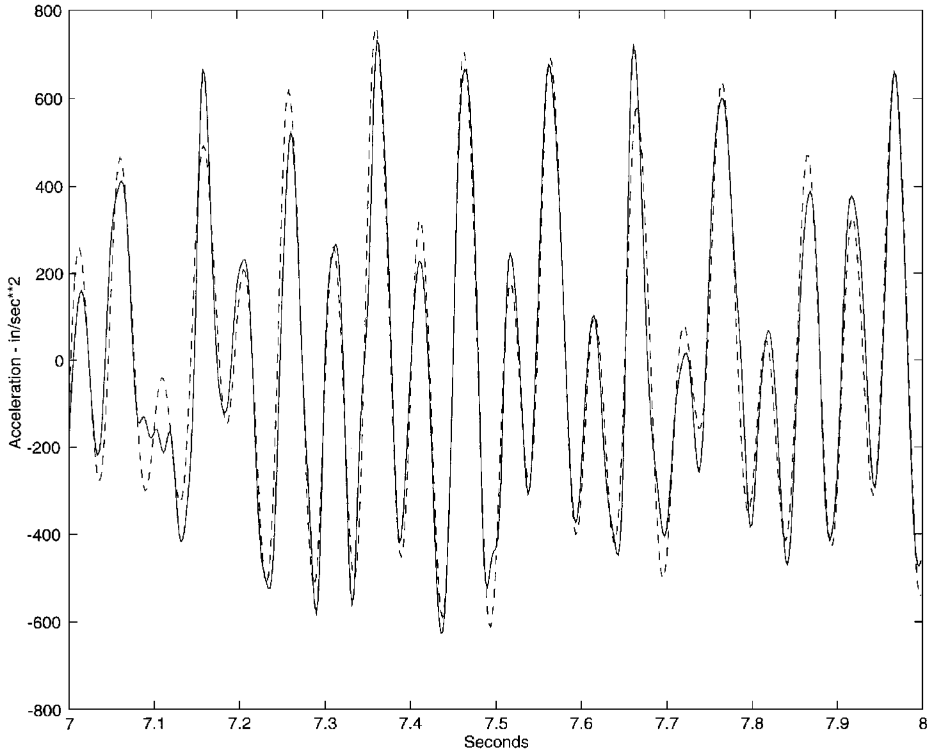

Fig. 7 Response at input location 140-z: – – –, exact and ——, predicted.

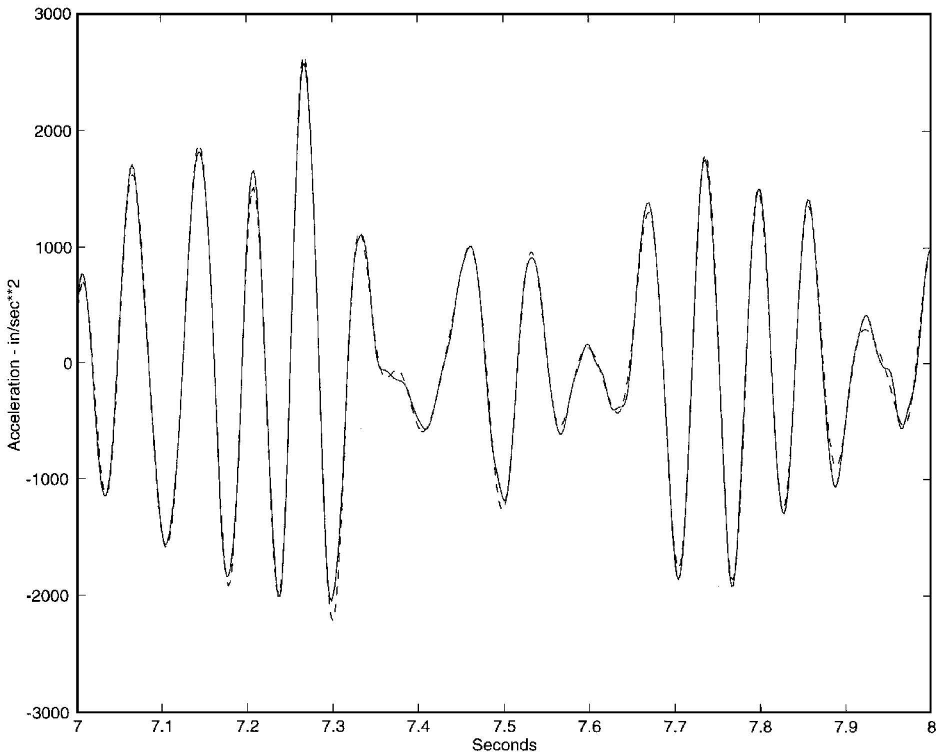

Fig. 8 Response at input location 310-x: – – –, exact and ——, predicted.

at both the sensor and the input locations. A time step of 0.002 s was employed, and 2.0% critical damping was assumed for all modes. During the simulation of the modal test 10% rms normally distributed noise with zero mean and unit standard deviation was added to the sensor data. Frequency response function matrices were generated using correlation functions with 20 averages. The Markov parameters were then computed using the inverse discrete Fourier transform. A total of 4096 data points were used in the analysis. A typical pulse response at node 294-x due to input at 3-y is shown in Fig. 2. The sensor and input location Markov parameters were assembled into the form of Eq. (24) and solved for the transformation P. Seventy block rows, N_R, and 600 block columns, N_C, were used to produce a transformation matrix with dimension (4×420). Selecting $N_R = 70$ implies that at the 69th time step and beyond, the current estimate of the input location response will depend on the current and past 69 measured sensor responses. In the presence of sensor noise, the system becomes approximately deadbeat at the 70th time step. Selecting $N_C = 600$ implies that only 600 time slices of data were used to compute P. Note from Fig. 2 that at the corresponding time of 1.2 s the pulse response has not yet decayed to zero.

Operation of the structure was simulated by applying external forces to the four selected input locations. Each of the simulated force time histories comprised 10 sinusoids with frequencies and amplitudes randomly selected between 0.0 and 20.0 Hz and 0.0 and 12.0 lb, respectively. No two input forces were the same. The acceleration response of the structure at the six sensor locations was measured for a duration of 4000 time steps, and 10% rms noise was again added to the measured sensor response. A typical noisy response time history is pictured in Fig. 3. The noisy sensor data were then passed through a low-pass filter with a cutoff frequency of 60.0 Hz. The filtered sensor data and the transformation matrix P were used in conjunction with Eq. (27) to estimate the response at the input locations over the duration of 4000 time steps. Figure 4 compares the predicted and exact response at input location 310-x. The prediction does a good job of matching the exact response over the entire 4000 time steps despite that only the first 600 time slices were used from the Markov parameters to compute transformation P. Prediction accuracy can be identified more easily by considering a smaller time slice. Figures 5–8 show the predicted vs exact response at all four input locations over the last 500 time steps. In each case, the predicted response is quite good. As expected, predictions are less accurate at input locations with smaller response levels due to a lower signal-to-noise ratio. In the absence of any sensor noise, the predicted responses were virtually exact.

Conclusion

A method has been presented for estimating the response of a structure during its operation at discrete locations that are inaccessible for measurement using sensors. The prediction is based on measuring response at other locations on the structure and transforming it into the response at the desired locations using a transformation matrix. The transformation is computed using the system Markov parameters determined from a vibration test in which the response is measured at both the locations that will possess sensors during structure operation and at the desired locations that will not possess sensors. Accelerometers were used as sensors throughout the analysis. Two different approaches were considered. The first required as many sensors as there are modes responding in the data. The second approach, a generalization of the first, only required as many sensors as the number of input locations. The desired response at the current time step is a function of the current and past measured responses. In both cases, sensors have to be placed such that the system is observable. The transformation matrix was shown to consist of a set of remote sensing system Markov parameters, which can be related to a set of inverse system observer Markov parameters. A numerical example was considered using the CEM testbed at NASA LaRC. A modal test and structural operation were simulated using a corresponding FEM. Acceleration response with 10% rms noise at six sensor locations was used to predict the response at four force input locations. The predictions were quite good considering the large noise levels. It is believed that the approach presented provides a valuable tool for predicting structural response with applications in many areas of structural dynamics, control, and identification. The proposed method is not computationally intensive. In fact, the transformation process itself is quite fast, consisting of only a matrix multiplication. This attribute, combined with the fact that the process is causal, may lead to real-time applications in the future.

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